Towards quantifying uncertainty in Greenland's contribution to 21st century sea-level rise

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Problem definition

Quantity of Interest in ice sheet modeling:

total ice mass loss/gain by, e.g. 2100 **→ sea level rise prediction**

Main sources of uncertainty:

- climate forcings (e.g. Surface Mass Balance)
- **basal friction**
- bedrock topography
- geothermal heat flux
- model parameters (e.g. Glen's Flow Law exponent)

Problem definition

Goal: Uncertainty Quantification of QoI

(Main) Issue: Huge number of parameters (10^5-10^7)

Work flow:

- Perform adjoint-based deterministic inversion to estimate initial ice sheet state. (i.e. characterize the present state of ice sheet to be used for performing prediction runs).
- Use deterministic inversion to build a Gaussian posterior in the inverse problem (based on recovered fields and the Hessian).
- Bayesian Calibration: construct the posterior distribution using Markov Chain Monte Carlo run on an emulator of the forward model.
- Forward Propagation: sample the obtained distribution and perform ensemble of forward propagation runs to compute the uncertainty on the QoI.

Deterministic Inversion

GOAL

Find ice sheet initial state that

- matches observations (e.g. surface velocity, temperature, etc.)
- matches present-day geometry (elevation, thickness)
- is in "equilibrium" with climate forcings (SMB)

by inverting for unknown/uncertain ice sheet model parameters.

Significantly reduce non physical transients without spin-up.

Bibliography

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- *Price, Payne, Howat and Smith*, PNAS, 2011
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- *Pollard DeConto*, TCD, 2012
- W. J. J.Van Pelt et al., The Cryosphere, 2013
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- *Goldberg and Heimbach*, The Cryosphere, 2013
- *Michel et al.*, Computers & Geosciences, 2014

Perego, Price, Stadler, **Journal of Geophysical Research**, 2014

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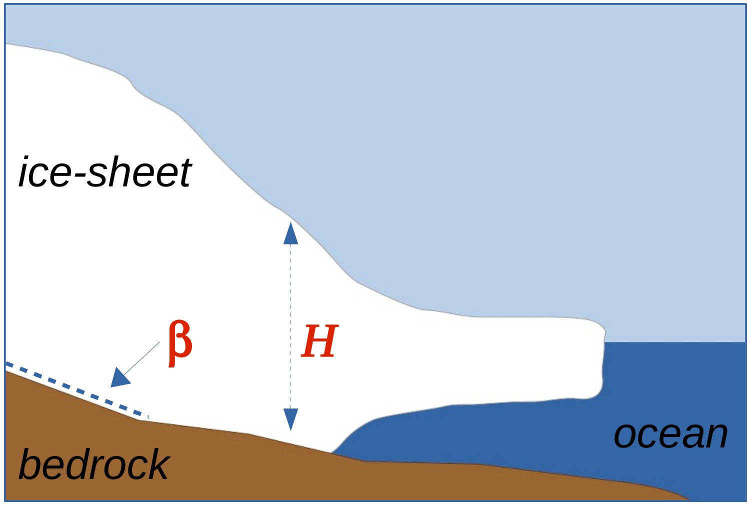
Deterministic Inversion

Problem details



Available data/measurements

- *ice extension and surface topography*
- ***surface velocity***
- ***Surface Mass Balance (SMB)***
- ***ice thickness H (sparse measurements)***

Fields to be estimated

- ***ice thickness H (allowed to vary but weighted by observational uncertainties)***
- ***basal friction β (spatially variable proxy for all basal processes)***

Modeling Assumptions

- *ice flow described by **nonlinear Stokes equation***
- *ice is close to **mechanical equilibrium***

Additional Assumption (for now)

- *given **temperature field***

Perego, Price, Stadler, **Journal of Geophysical Research**, 2014

Deterministic Inversion

PDE-constrained optimization problem: cost functional

Problem: find initial conditions such that the ice is close to thermo-mechanical equilibrium, given the geometry and the SMB, and matches available observations.

Optimization problem:

find β and H that minimizes the functional $\mathcal{J}$

$$
\begin{aligned}
\mathcal{J}(\beta, H) &= \int_\Sigma \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds && \text{surface velocity mismatch} \\
&+ \int_\Sigma \frac{1}{\sigma_\tau^2} |\mathrm{div}(\boldsymbol{U} H) - \tau_s|^2 \, ds && \text{SMB mismatch} \\
&+ \int_\Sigma \frac{1}{\sigma_H^2} |H - H^{obs}|^2 \, ds && \text{thickness mismatch} \\
&+ \mathcal{R}(\beta, H) && \text{regularization terms.}
\end{aligned}
$$

Common

Proposed

subject to ice sheet model equations (FO or Stokes)

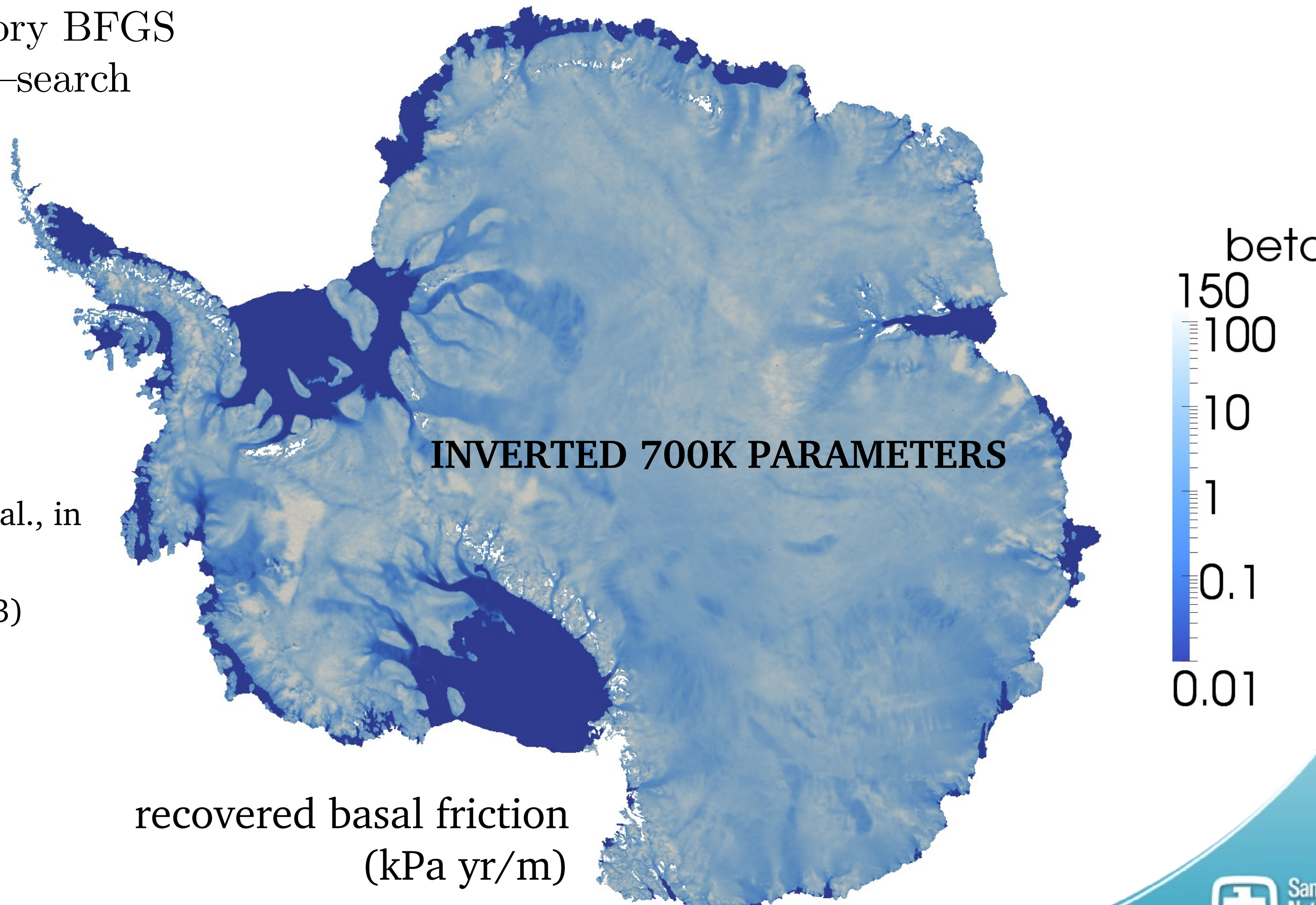
$\boldsymbol{U}$: computed depth averaged velocity
H: ice thickness
β: basal sliding friction coefficient
τ_s: SMB
$\mathcal{R}(\beta)$ regularization term

Antarctica Inversion (only for basal friction)

Objective functional: $\mathcal{J}(\mathbf{u}(\beta),\ \beta) = \int_\Sigma \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds + \ \alpha \int_\Sigma |\nabla \beta|^2 \, ds$

ROL algorithm:

- Limited–Memory BFGS
- Backtrack line–search



recovered basal friction (kPa yr/m)

Gometry (Cornford, Martin et al., in prep.)

Bedmap2 (Fretwell et al., 2013)

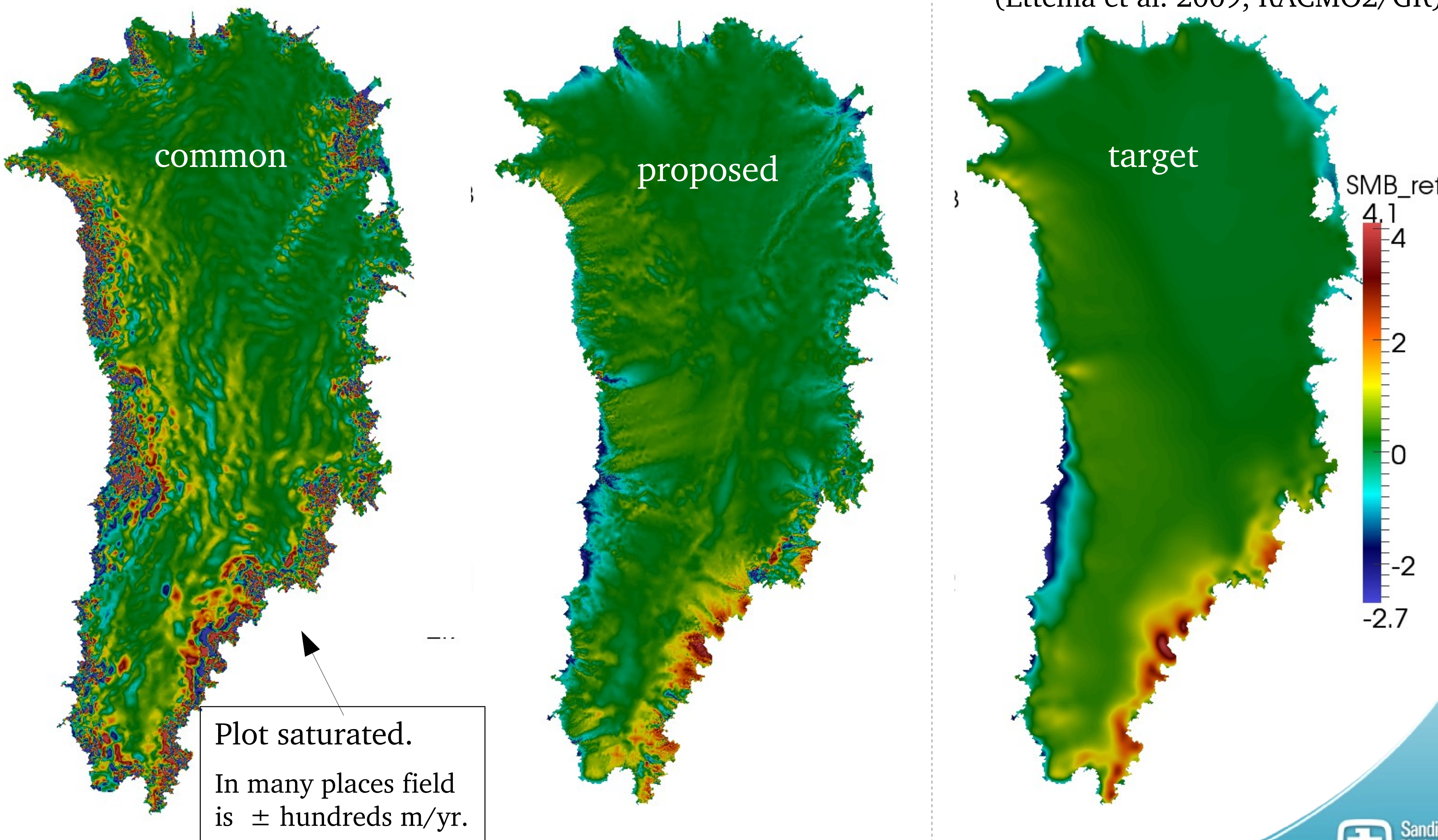
Temperature (Pattyn, 2010)

Deterministic Inversion for Greenland ice sheet

Inversion results: surface mass balance (SMB)

SMB (m/yr) needed for equilibrium

SMB from climate model
(Ettema et al. 2009, RACMO2/GR)



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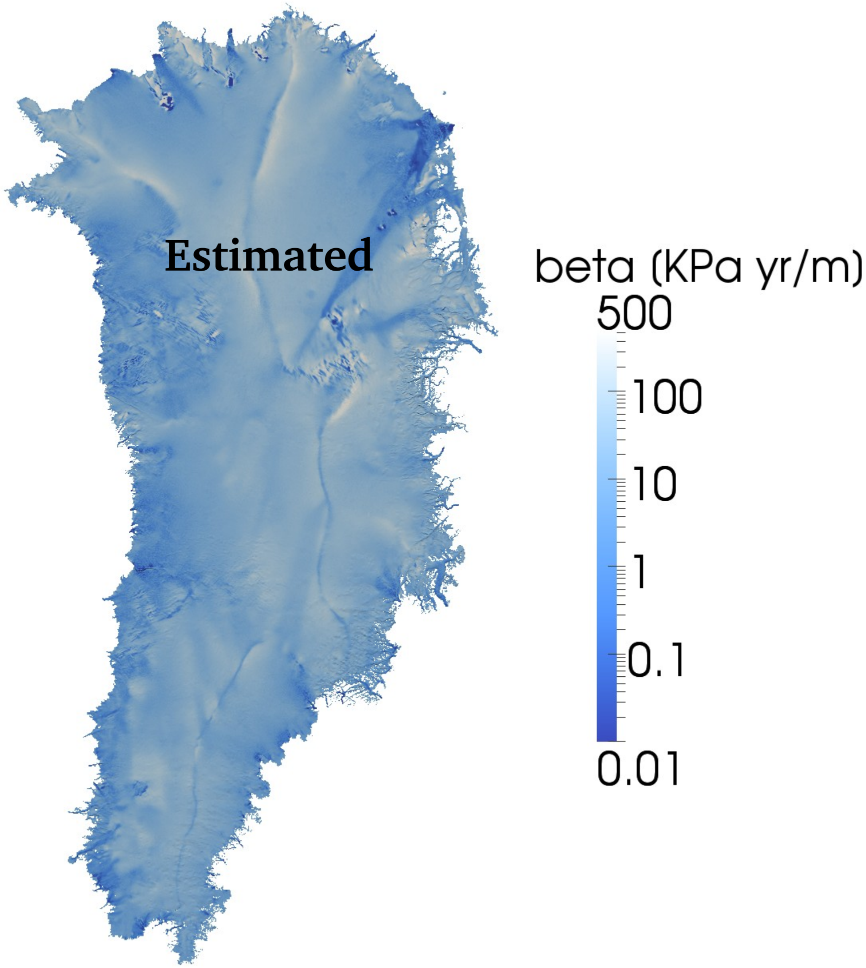
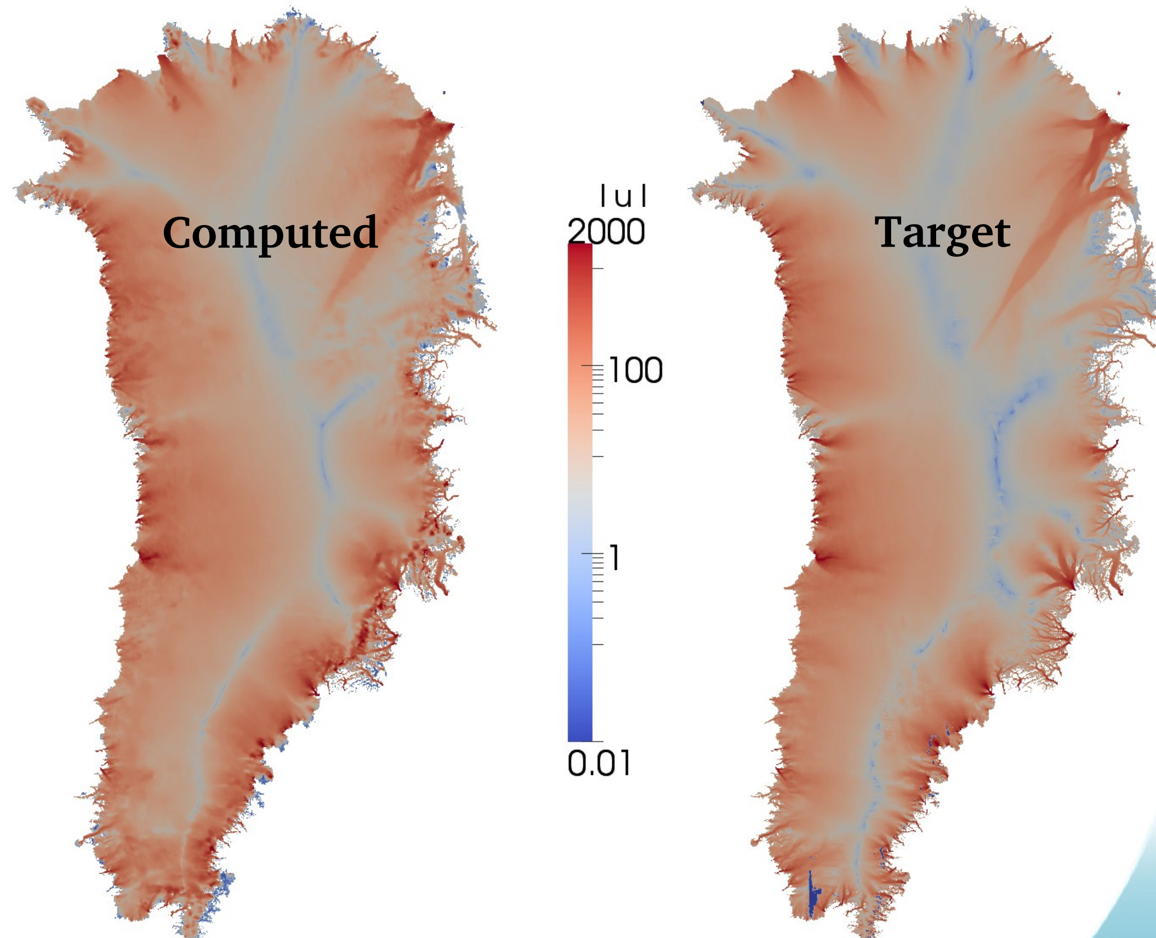
Greenland Inversion using Albany-Piro-ROL

Inversion with 1.6M parameters



Basal friction coefficient (m/yr)



surface velocity magnitude (m/yr)

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Bayesian Calibration (proof of concept w/ KLE)

Difficulty in UQ approach: *"Curse of dimensionality"*.
At relevant model resolutions, the basal friction parameter space can have $O(10^6)$ parameters.
However, the effective dimension of the problem is smaller.

1. Assume analytic covariance kernel $\Gamma_{\text{prior}} = \exp\left(-\frac{|r_1 - r_2|^2}{L^2}\right)$. First attempt, we intend to use Hessian based covariance in the future.

2. Perform eigenvalue decomposition of Γ_{prior}.

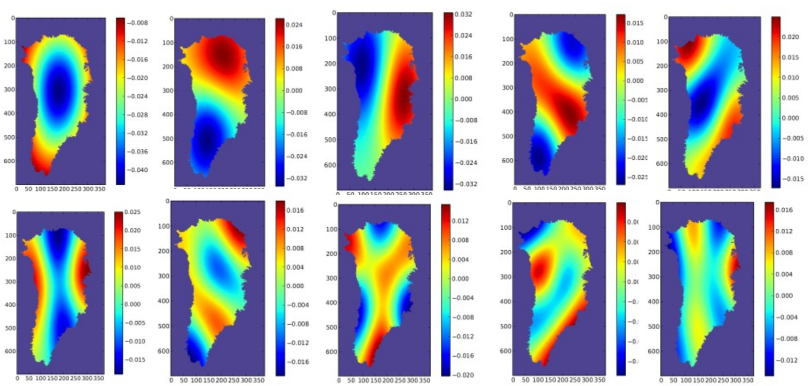
3. Take the mean $\bar{\beta}$ to be the deterministic solution and expand β in basis of eigenvector $\{\phi_k\}$ of Γ_{prior}, with random variables $\{\xi_k\}$

$$\beta(\omega) = \bar{\beta} + \sum_{k=1}^{K} \sqrt{\lambda_k}\phi_k\xi_k(\omega)$$

*Expansion done on $\log(\beta)$ to avoid negative values for β.

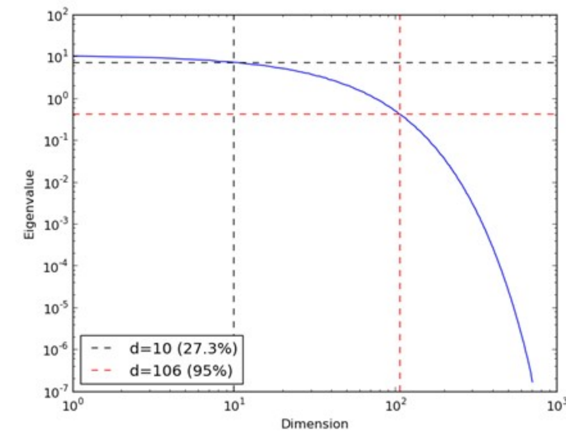
Bayesian Calibration and Uncertainty Propagation
(feasibility study)

- First 10 KLE modes
  (parallel C++/Trilinos code **Anasazi**).



Only spatial correlation has been considered.

Eigenvalues Decay
(100 eigenvalues capture 95% energy)



- Mismatch (**ALBANY**): $\mathcal{J}(\beta) = \int_{\Sigma} \frac{1}{\sigma_u^2} |\mathbf{u}(\beta) - \mathbf{u}^{obs}|^2 + \alpha |\nabla \beta|^2$

- ***Build Emulator.*** Polynomial chaos expansion (**PCE**) was formed for the mismatch over random variables with uniform prior distributions using almost 300 steady-state simulations. **DAKOTA**.

Emulator (Polynomial Chaos Expansion): $\beta(\omega) = \bar{\beta} + \sum_{k=1}^{K} \sqrt{\lambda_k} \phi_k \xi_k(\omega)$

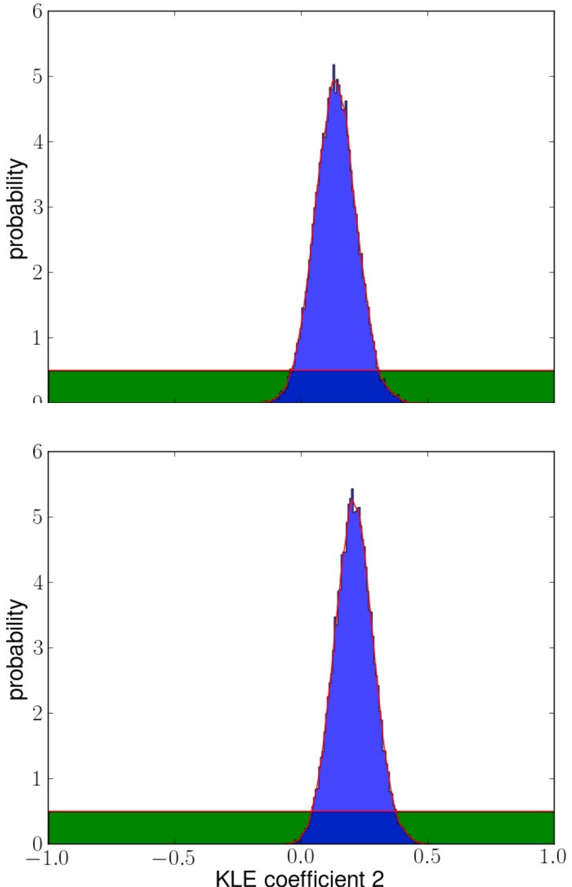
Bayesian Calibration and Uncertainty Propagation

(feasibility study)

- **Inversion/Calibration**. Markov Chain Monte Carlo (**MCMC**),delayed rejection adaptive metropolis (DRAM), was performed on the PCE **QUESO**.

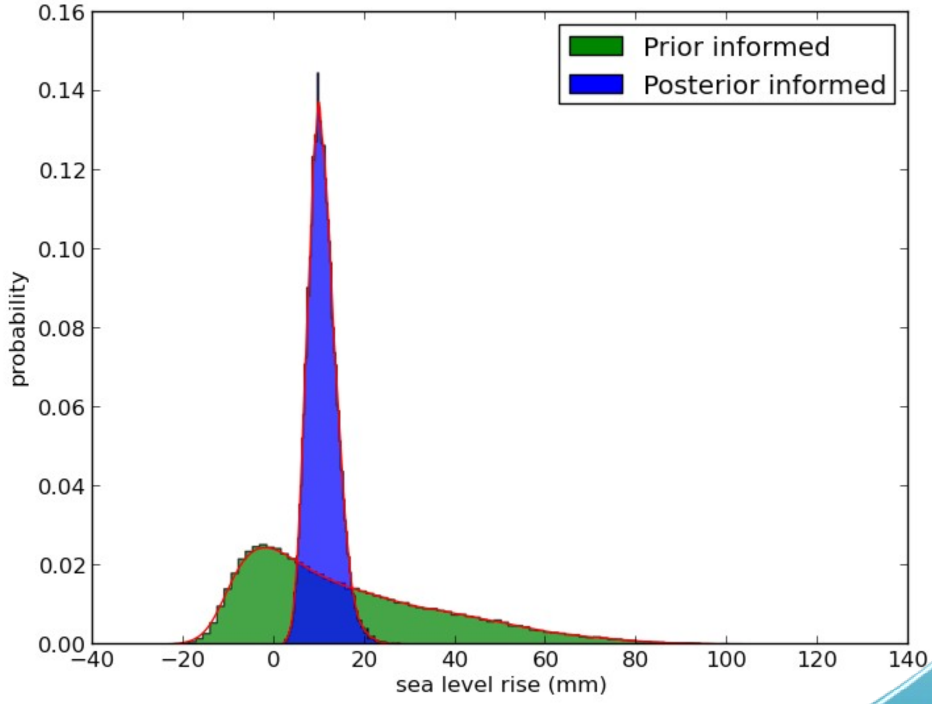
- **Uncertainty propagation**. Used Gaussian process to build surrogate using 66 transient simulations.

Posterior distributions



Uncertainty propagation:

Sea level rise prediction in 50 years



Bayesian Calibration and Uncertainty Propagation
(discussion on feasibility study)

- Prior chosen is somewhat arbitrary, however it is possible to build an informed Gaussian distribution using Hessian of the deterministic inversion.
- Prior distribution size is big (in real application million of parameters with thousands significant parameters) and so the KLE expansion needs several modes to retain most of the prior energy – in the results shown we only retained 27% of the prior energy!
- A lot of samples are needed to build the emulator. Cross correlation tests showed that the emulator we built for the uncertainty propagation was not sufficient for building the emulator.
- We might use techniques such as **compressed sensing technique*** to adaptively select significant modes and the basis for the parameter space. The hope is that only few modes affect the low dimensional QoI (e.g. sea level rise).
- It might be to use cheap physical models (e.g. SIA) or low resolution solves to reduce the cost of building the emulator.

*Jakeman, Eldred, Sargsyan, JCP, 2015

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Thank you!